| **Required Course Numbers** | | | | | | | | | | | | | | | |
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| **Test Content Categories** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **I. Number and Quantity, Algebra, Functions, and Calculus (68%)** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **A. Number and Quantity** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understand the properties of exponents |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. perform operations involving exponents, including negative and rational exponents |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. demonstrate an understanding of the properties of exponential expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use the properties of exponents to rewrite expressions that have radicals or rational exponents |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understand the properties of rational and irrational numbers, and the interactions between those sets of numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. recognize that the sum or product of two rational numbers is rational |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. recognize that the sum of a rational number and an irrational number is irrational |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. recognize that the product of a nonzero rational number and an irrational number is irrational |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. recognize that the sum or product of two irrational numbers can be rational or irrational |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understand how to solve problems by reasoning quantitatively (e.g., dimensional analysis, reasonableness of solutions) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use units as a way to understand problems and to guide the solution of multistep problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. choose and interpret units consistently in formulas |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. choose and interpret the scale and the origin in graphs and data displays |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. recognize the reasonableness of results within the context of a given problem |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understand the structure of the natural, integer, rational, real, and complex number systems and how the basic operations (+, –, ×, and ÷) on numbers in these systems are performed |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. solve problems using addition, subtraction, multiplication, and division of rational, irrational, and complex numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. apply the order of operations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. given operations on a number system, determine whether the properties (e.g., commutative, associative, distributive) hold |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. compare, classify, and order real numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. simplify and approximate radicals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. find conjugates of complex numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. demonstrate an understanding of the properties of counting numbers (e.g., prime, composite, prime factorization, even, odd, factors, multiples) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understand how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. solve quadratic equations with real coefficients that have complex solutions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. extend polynomial identities to the complex numbers (e.g., ) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. verify the fundamental theorem of algebra for quadratic polynomials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understand how to perform operations on matrices and how to use matrices in applications |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use matrices to represent and manipulate data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. multiply matrices by scalars to produce new matrices |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. add, subtract, and multiply matrices of appropriate dimensions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. understand that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. understand the role played by zero and identity matrices in matrix addition and multiplication |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. understand that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understand how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. apply the concept of a ratio and use ratio language and notation to describe a relationship between two quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. compute unit rates |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use ratio reasoning to convert rates |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. solve problems involving scale factors |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. recognize and represent proportional and inversely proportional relationships between two quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. use proportional relationships to solve multistep ratio, average, and percent problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. solve measurement and estimation problems involving time, length, temperature, volume, and mass in both the U.S. customary system and the metric system, where appropriate |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. convert units within the metric and U.S. customary systems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Know how to analyze both precision and accuracy in measurement situations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. choose a level of accuracy appropriate to limitations on measurement when reporting quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. calculate or estimate absolute and relative error in the numerical answer to a problem |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Understand various ways to represent and compare very large and very small numbers (e.g., scientific notation, orders of magnitude) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. represent and compare very large and very small numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10. Understand how to both estimate and perform calculations on very large and very small quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use orders of magnitude to estimate very large and very small numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. perform calculations on numbers in scientific notation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **B. Algebra** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understand how to write algebraic expressions in equivalent forms |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use the structure of an expression to identify ways to rewrite it |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand how to rewrite quadratic expressions for specific purposes (e.g., factoring/finding zeros, completing the square/finding maxima or minima) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use the properties of exponents to rewrite expressions for exponential functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understand how to perform arithmetic operations on polynomials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. add, subtract, and multiply polynomials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understand the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. know and apply the remainder theorem: for a polynomial  and a number *a*, the remainder on division by  is  , so  if and only if  is a factor of |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use factorization to identify zeros of polynomials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use zeros of a polynomial to construct a rough graph of the function defined by the polynomial |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understand how to use polynomial identities (e.g., difference of squares, sum and difference of cubes) to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. apply the binomial theorem for the expansion of  in powers of *x* and *y* for a positive integer *n* |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understand how to rewrite rational expressions and perform arithmetic operations on rational expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. rewrite simple rational expressions in different forms |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. add, subtract, multiply, and divide rational expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understand how to create equations and inequalities that describe relationships |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. create equations and inequalities in one variable and use them to solve problems and graph solutions on the number line |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. create equations and inequalities in two or more variables to represent relationships between quantities, solve problems, and graph them on the coordinate plane with labels and scales |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. represent constraints by equations, inequalities, or systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. rearrange formulas to highlight a quantity of interest (e.g., solve  for *t*) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understand how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. explain each step in solving a simple equation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. solve simple rational and radical equations in one variable, incorporating analysis of possible extraneous solutions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understand how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities in one variable |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. solve linear equations and inequalities in one variable, including equations with coefficients represented by letters |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use the method of completing the square to transform any quadratic equation in *x* into the equivalent form |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. solve equations using a variety of methods (e.g., using graphs, using the quadratic formula, factoring) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. use different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. write complex solutions in the form |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Understand how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. explain why, when solving a system of two equations using the elimination method, replacing one or both equations with a scalar multiple produces a system with the same solutions as the solutions of the original system |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. solve a system consisting of two linear equations in two variables algebraically and graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. represent a system of linear equations as a single matrix equation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. find the inverse of a matrix, if it exists, and use it to solve systems of linear equations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. explain why the *x*-coordinates of the intersection points of the graphs of  and  are the solutions of |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. find the solutions of  approximately (e.g., use technology to graph the functions, make tables of values, find successive approximations); include cases where  and/or  are linear, polynomial, rational, absolute value, exponential, or logarithmic functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding halfplanes |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10. Understand the properties of number systems under various operations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. given operations on algebraic expressions, determine whether the properties hold (e.g., commutative, associative, distributive) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11. Understand the concept of rate of change of nonlinear functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. calculate and interpret the average rate of change of a function presented symbolically, numerically, or graphically over a specified interval |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12. Understand the concepts of intercept(s) of a line and slope as a rate of change |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. calculate and interpret the intercepts of a line |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. calculate and interpret the slope of a line presented symbolically, numerically, or graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. estimate the rate of change of a linear function from a graph |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 13. Understand how to find the zero(s) of functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use a variety of techniques to find and analyze the zero(s) (real and complex) of functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **C. Functions** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understand the function concept and the use of function notation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. recognize that functions are sets of ordered pairs |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use function notation, evaluate functions, and interpret statements that use function notation in terms of a context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understand how to find the domain and range of a function and a relation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. identify the domain and range of a function or relation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. determine the domain of a function from a function rule (e.g., ), graph, set of ordered pairs, or table |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understand how function behavior is analyzed using different representations (e.g., graphs, mappings, tables) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. interpret key features of graphs and tables (e.g., increasing/decreasing, maximum/minimum, periodicity) in terms of the quantities for a function that models a relationship between two quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. given a verbal description of a relation, sketch graphs that show key features of that relation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. graph functions (i.e., radical, piecewise, absolute value, polynomial, rational, logarithmic, trigonometric) expressed symbolically and identify key features of the graph |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. write a function that is defined by an expression in different but equivalent forms to reveal different properties of the function (e.g., zeros, extreme values, symmetry of the graph) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. interpret the behavior of exponential functions (e.g., growth, decay) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. understand how to determine if a function is odd, even, or neither, and any resulting symmetries |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understand how functions and relations are used to model relationships between quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. write a function that relates two quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. determine an explicit expression or a recursive process that builds a function from a context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understand how new functions are obtained from existing functions (e.g., compositions, transformations, inverses) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. describe how the graph of  is related to the graph of , where , , , , or  for specific values of *k* (both positive and negative), and find the value of *k* given the graphs  b. determine if a function has an inverse and, if so, write an expression for the inverse |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. verify by composition if one function is the inverse of another |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. given that a function *f* has an inverse, find values of the inverse function from a graph or a table of *f* |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. given a noninvertible function, determine a largest possible domain of the function that produces an invertible function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. understand the inverse relationship between exponential and logarithmic functions and use this relationship to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. combine standard function types using arithmetic operations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. perform domain analysis on functions resulting from arithmetic operations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| i. compose functions algebraically, numerically, and graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| j. perform domain analysis on functions resulting from compositions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understand differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. recognize situations in which one quantity changes at a constant rate per unit interval relative to another |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (including reading these from a table) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. express the solution to an exponential equation with base *b* as a logarithm (e.g., |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. use technology to evaluate logarithms that have any base |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. interpret the parameters in a linear or exponential function in terms of a context  (e.g.,  ) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| i. use quantities that are inversely related to model phenomena |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understand how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. understand radian measure (e.g., one radian is the measure of a central angle that subtends an arc with length equal to the length of the radius) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand how the domains of trigonometric functions can be extended beyond 0 to  using the unit circle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use special triangles (i.e., 30-60-90, 45-45-90) to determine geometrically the values of sine, cosine, and tangent for |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. use reference angles to find the values of trigonometric functions at angles outside the interval 0 to |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. use the unit circle to explain symmetry and periodicity of trigonometric functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understand how periodic phenomena are modeled using trigonometric functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand how to restrict the domain of a trigonometric function so that its inverse can be constructed |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use inverse functions to solve trigonometric equations that arise in modeling contexts, and interpret them in terms of the context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Understand the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use Pythagorean identities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use trigonometric identities to rewrite expressions and solve equations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. understand trigonometric identities in the context of equivalent graphs of trigonometric functions (e.g.,  and  are equivalent graphs) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. prove Pythagorean identities  (e.g.  ) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10. Know how to interpret representations of functions of two variables (e.g., three- dimensional graphs, tables) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. interpret representations of functions of two variables |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11. Understand how to solve equations (e.g., trigonometric, logarithmic, exponential) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. solve trigonometric, logarithmic, and exponential equations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **D. Calculus** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understand the meaning of a limit of a function and how to calculate limits of functions, determine when the limit does not exist, and solve problems using the properties of limits |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. graphically analyze the limit of  as *x* approaches a fixed value from both left and right |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. solve limit problems (e.g., a constant times a function, the sum of two functions, the product and quotient of two functions) using properties of limits, where all limits of the individual functions exist at the value that *x* is approaching |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. analyze one-sided limits for various functions to see whether or not the limit exists |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. recognize limits that do not exist, such as: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understand the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. construct a function graph for a given function and a given point , and explain what happens to the succession of slopes of secant lines connecting  to  as *x* approaches *a*, from both the right side and the left side |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. state the limit definition of the derivative, and use it to find the derivative of a given function at a given value of *x* and to find the derivative function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understand how to show that a particular function is continuous  a. apply the three steps (i.e.,  exists,  exists, and  ) that are part of the definition of what it means for a function to be continuous at *x* = *a* to verify whether a given function is continuous at a given point |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Know the relationship between continuity and differentiability |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. give examples of functions that are continuous at *x* = *a* but not differentiable at *x* = *a*, and explain why |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understand how to approximate derivatives and integrals numerically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. given a table of values, use the slope of a secant line to approximate a derivative |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use the midpoint rule, trapezoid rule, or other Reimann sums to find numerical approximations for integrals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understand how and when to use standard differentiation and integration techniques |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use standard differentiation techniques |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use standard integration techniques |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. understand the relationship between position, velocity, and acceleration functions of a particle in motion |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understand how to analyze the behavior of a function (e.g., extrema, concavity, symmetry) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use the first and second derivatives to analyze the graph of a function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understand how to apply derivatives to solve problems (e.g., related rates, optimization) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. apply derivatives to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Understand the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. solve problems using the foundational theorems of calculus |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand the relationship between differentiation and integration, including the role of the fundamental theorems of calculus |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. match graphs of functions with graphs of their derivatives or accumulations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. understand how to use differentiation and integration of a function to express rates of change and total change |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. understand and calculate the average value of a function over an interval (i.e., mean value theorem of integrals) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10. Understand integration as a limit of Riemann sums |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. calculate a definite integral using a limit of Riemann sums |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11. Understand how to use integration to compute area, volume, distance, or other accumulation processes |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use integration techniques to compute area, volume, distance, or other accumulation processes |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12. Know how to determine the limits of sequences, if they exist |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. determine the limits of sequences, when they exist |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 13. Is familiar with simple infinite series |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. determine if simple infinite series converge or diverge |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. find the sum of a simple infinite series, if it exists |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. find the partial sum of a simple infinite series |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **II. Geometry, Probability and Statistics, and Discrete Mathematics (32%)** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **A. Geometry** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understand transformations in a plane |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. know precise definitions of angle, circle, line segment, perpendicular lines, and parallel lines |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. represent transformations in the plane |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. describe transformations as functions that take points in the plane as inputs, and give other points as outputs |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. recognize whether a transformation preserves distance and angle measure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. given a geometric figure and a rotation, reflection, or translation, draw the transformed figure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. specify a sequence of transformations that will map a given figure onto another figure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understand how to prove geometric theorems such as those about lines and angles, triangles, and parallelograms |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. prove theorems about lines and angles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. prove theorems about triangles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. prove theorems about parallelograms |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understand how geometric constructions are made with a variety of tools and methods |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. recognize formal geometric constructions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. explain how formal geometric constructions are made (e.g., an equilateral triangle, a square, a regular hexagon inscribed in a circle) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understand congruence and similarity in terms of transformations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. verify the properties of dilations given by a center and a scale factor |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. given two figures, use the definition of similarity in terms of dilations to decide if the figures are similar |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. explain how the criteria for triangle congruence (e.g., ASA, SAS, HL) follow from the definition of congruence in terms of rigid motions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. use the properties of similarity transformations to establish the AA criterion for two triangles to be similar |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understand how trigonometric ratios are defined in right triangles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. explain and use the relationship between the sine and cosine of complementary angles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understand how trigonometry is applied to general triangles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. derive the formula  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side and use it to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. apply the law of sines and the law of cosines to find unknown measurements in triangles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understand and applies theorems about circles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. identify and describe relationships among inscribed angles, radii, and chords |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. prove properties of angles for a quadrilateral inscribed in a circle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. construct a tangent line from a point outside a given circle to the circle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understand arc length and area measurements of sectors of circles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. derive and use the fact that the length of the arc intercepted by a central angle is proportional to the circumference |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. derive and use the formula for the area of a sector |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Know how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. derive and use the equation of a circle of given center and radius |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. complete the square to find the center and radius of a circle given by an equation in standard form |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. derive the equation of a parabola given a focus and directrix |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. derive and use the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from a point on the curve to the foci is constant |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10. Understand how to use coordinate geometry to algebraically prove simple geometric theorems  a. use coordinates to prove simple geometric theorems algebraically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. prove the slope criteria for parallel and perpendicular lines, and use parallel and perpendicular lines to solve geometric problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. find the point on a directed line segment between two given points that partitions the segment in a given ratio |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. use coordinates to compute perimeters of polygons and areas of triangles and quadrilaterals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11. Understand how perimeter, area, surface area, and volume formulas are used to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. give an informal argument for the formulas for the circumference of a circle, the area of a circle, and the volume of a cylinder, pyramid, and cone |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use the perimeter and area of geometric shapes to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use the surface area and volume of prisms, cylinders, pyramids, cones, and spheres to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12. Know how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use two-dimensional representations of three-dimensional objects to visualize and solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 13. Know how to apply geometric concepts in real-world situations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use geometric shapes, their measures, and their properties to describe objects |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. apply concepts of density based on area and volume in modeling situations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. apply geometric methods to solve design problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 14. Understand the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. solve problems involving parallel, perpendicular, and intersecting lines |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. apply angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. solve problems that involve medians, midpoints, and altitudes |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. solve problems involving special triangles (e.g., isosceles, equilateral, right) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. know geometric properties of various quadrilaterals (e.g., parallelograms, trapezoids) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. know relationships among quadrilaterals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. solve problems involving angles and diagonals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. solve problems involving polygons with more than four side |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **B. Probability and Statistics** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understand how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., box plots, dot plots, normal distributions) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. represent data with plots on the real number line (e.g., dot plots, histograms, and box plots) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, and recognize that there are data sets for which such a procedure is not appropriate |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. estimate areas under the normal curve |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understand how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. summarize and interpret categorical data for two categories in two-way frequency tables (e.g., joint, marginal, conditional relative frequencies) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. recognize possible associations and trends in the data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. represent data for two quantitative variables on a scatterplot, and describe how the variables are related |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understand how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use technology to fit a function to data (i.e., linear regression) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use functions fitted to data to solve problems in the context of the data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. assess the fit of a function by plotting and analyzing residuals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. interpret the slope and the intercept of a regression line in the context of the data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. compute and interpret a correlation coefficient |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. distinguish between correlation and causation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understand statistical processes and how to evaluate them |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. understand statistics as a process for making inferences about population parameters based on a random sample from that population |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understand how to make inferences and justify conclusions from samples, experiments, and observational studies |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. recognize the purposes of and differences among samples, experiments, and observational studies, and explain how randomization relates to each |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use data from a sample to estimate a population mean or proportion |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use data from a randomized experiment to compare two treatments |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. use results of simulations to decide if differences between parameters are significant |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. evaluate reports based on data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understand the concepts of independence and conditional probability and how to apply these concepts to data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand that two events, *A* and *B*, are independent if and only if    c. understand the conditional probability of A given B as  , and interpret independence of *A* and *B* as saying that  and |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. recognize and explain the concepts of conditional probability and independence |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. calculate probabilities of simple and compound events |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. construct and interpret two-way frequency tables of data when two categories are associated with each object being classified; use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. find  and interpret it in terms of a given model |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. apply the addition rule, , and interpret it in terms of a given model |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. apply the general multiplication rule in a uniform probability model, ,  and interpret it in terms of a given model |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. calculate probabilities using the binomial probability distribution |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Know how to make informed decisions using probabilities and expected values |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, and graph the corresponding probability distribution using the same graphical displays as for data distributions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. calculate the expected value of a random variable, and interpret it as the mean of the probability distribution |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. develop a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and find the expected value |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. develop a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and find the expected value |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. analyze decisions and strategies using probability concepts (e.g., fairness) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Understand how to use simulations to construct experimental probability distributions and to make informal inferences about theoretical probability distributions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. given the results of simulations, construct experimental probability distributions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. given the results of simulations, make informal inferences about theoretical probability distributions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10. Understand how to find probabilities involving finite sample spaces and independent trials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use the fundamental counting principle to find probabilities involving finite sample spaces and independent trials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **C. Discrete Mathematics** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understand sequences (e.g., arithmetic, recursively defined, geometric) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. evaluate, extend, or algebraically represent rules that involve number patterns |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. explore patterns in order to make conjectures, predictions, or generalizations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Is familiar with how recursion can be used to model various phenomena |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. find values of functions defined recursively, and understand how recursion can be used to model various phenomena |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. convert between recursive and closed-form expressions for a function, where possible |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Has knowledge of equivalence relations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. determine whether a binary relation on a set is reflexive, symmetric, or transitive |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. determine whether a relation is an equivalence relation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understand the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. understand the differences between discrete and continuous representations (e.g., data, functions) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand how discrete and continuous representations can be used to model various phenomena |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understand basic terminology and symbols of logic |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. understand the basic terminology of logic |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. understand the symbols of logic |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. use logic to evaluate the truth of statements |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. use logic to evaluate the equivalence of statements (e.g., statement and contrapositive) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understand how to use counting techniques such as the multiplication principle, permutations, and combinations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. use counting techniques to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understand basic set theory (e.g., unions, differences, Venn diagrams) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. solve problems using basic set theory (i.e., union, intersection, complement, difference) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. use Venn diagrams to answer questions about sets |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |