GRADUATE RECORD EXAMINATIONS®

Official GRE Quantitative Reasoning Problem-Solving Strategies

Large Print (18 point) Edition

General Problem-Solving Steps

Questions in the Quantitative Reasoning measure ask you to model and solve problems using quantitative, or mathematical, methods. Generally, there are three basic steps in solving a mathematics problem:

Step 1: Understand the problem
Step 2: Carry out a strategy for solving the problem
Step 3: Check your answer

Here is a description of the three steps, followed by a list of useful strategies for solving mathematics problems.

Step 1: Understand the Problem

The first step is to read the statement of the problem carefully to make sure you understand the information given and the problem you are being asked to solve.

Some information may describe certain quantities. Quantitative information may be given in words or mathematical expressions, or a combination of both. Also, in some problems you may need to read and understand quantitative information in data presentations, geometric figures, or coordinate systems. Other information may take the form of formulas, definitions, or conditions that must be satisfied by the quantities. For example, the conditions may be equations or inequalities, or may be words that can be translated into equations or inequalities.

In addition to understanding the information you are given, it is important to understand what you need to accomplish in order to solve the problem. For example, what unknown quantities must be found? In what form must they be expressed?

Step 2: Carry Out a Strategy for Solving the Problem

Solving a mathematics problem requires more than understanding a description of the problem, that is, more than understanding the quantities, the data, the conditions, the unknowns, and all other mathematical facts related to the problem. It requires determining <u>what</u> mathematical facts to use and <u>when</u> and <u>how</u> to use those facts to develop a solution to the problem. It requires a strategy.

Mathematics problems are solved by using a wide variety of strategies. Also, there may be different ways to solve a given problem. Therefore, you should develop a repertoire of problem-solving strategies, as well as a sense of which strategies are likely to work best in solving particular problems. Attempting to solve a problem without a strategy may lead to a lot of work without producing a correct solution.

After you determine a strategy, you must carry it out. If you get stuck, check your work to see if you made an error in your solution. It is important to have a flexible, open mind-set. If you check your solution and cannot find an error or if your solution strategy is simply not working, look for a different strategy.

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Step 3: Check Your Answer

When you arrive at an answer, you should check that it is reasonable and computationally correct.

- Have you answered the question that was asked?

- Is your answer reasonable in the context of the question? Checking that an answer is reasonable can be as simple as recalling a basic mathematical fact and checking whether your answer is consistent with that fact. For example, the probability of an event must be between 0 and 1, inclusive, and the area of a geometric figure must be positive. In other cases, you can use estimation to check that your answer is reasonable. For example, if your solution involves adding three numbers, each of which is between 100 and 200, estimating the sum tells you that the sum must be between 300 and 600.

- Did you make a computational mistake in arriving at your answer? A key-entry error using the calculator? You can check for errors in each step in your solution. Or you may be able to check directly that your solution is correct. For example, if you solved the equation $7(3x - 2) + 4 = 95$ for x and got the answer $x = 5$, you can check your answer by substituting $x = 5$ into the equation to see that $7(3(5) - 2) + 4 = 95$.

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Strategies

There are no set rules—applicable to all mathematics problems—to determine the best strategy. The ability to determine a strategy that will work grows as you solve more and more problems. What follows are brief descriptions of useful strategies. Along with each strategy one or two sample questions that you can answer with the help of the strategy are given. These strategies do not form a complete list, and, aside from grouping the first four strategies together, they are not presented in any particular order.

The first four strategies are translation strategies, where one representation of a mathematics problem is translated into another.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

Word problems are often solved by translating textual information into an arithmetic or algebraic representation. For example, an "odd integer" can be represented by the expression $2n + 1$, where n is an integer; and the statement "the cost of a taxi trip is \$3.00, plus \$1.25 for each mile" can be represented by the expression $c = 3 + 1.25m$. More generally, translation occurs when you understand a word problem in mathematical terms in order to model the problem mathematically.

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Sample Question 1 for Strategy 1: Multiple-Choice – Select One Answer Choice Question.

A car got 33 miles per gallon using gasoline that cost $2.95 per gallon. Approximately what was the cost, in dollars, of the gasoline used in driving the car 350 miles?

(A) $10

(B) $20

(C) $30

(D) $40

(E) $50

Explanation

Scanning the answer choices indicates that you can do at least some estimation and still answer confidently. The car used $\dfrac{350}{33}$ gallons of gasoline, so the cost was $\left(\dfrac{350}{33}\right)(2.95)$ dollars. You can estimate the product $\left(\dfrac{350}{33}\right)(2.95)$ by estimating $\dfrac{350}{33}$ a little low, 10, and estimating 2.95 a little high, 3, to get approximately $(10)(3) = 30$ dollars. You can also use the calculator to compute a more exact answer and then round the answer to the nearest 10 dollars, as suggested by the answer choices. The calculator yields the decimal 31.287…, which rounds to 30 dollars. **Thus, the correct answer is Choice C, $30**.

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Sample Question 2 for Strategy 1: Numeric Entry Question.

Working alone at its constant rate, machine *A* produces *k* liters of a chemical in 10 minutes. Working alone at its constant rate, machine *B* produces *k* liters of the chemical in 15 minutes. How many minutes does it take machines *A* and *B*, working simultaneously at their respective constant rates, to produce *k* liters of the chemical?

−	.	.	.	.	.	.	.
	0	0	0	0	0	0	0
	1	1	1	1	1	1	1
	2	2	2	2	2	2	2
	3	3	3	3	3	3	3
	4	4	4	4	4	4	4
	5	5	5	5	5	5	5
	6	6	6	6	6	6	6
	7	7	7	7	7	7	7
	8	8	8	8	8	8	8
	9	9	9	9	9	9	9

minutes

Explanation

Machine A produces $\dfrac{k}{10}$ liters per minute, and machine B produces $\dfrac{k}{15}$ liters per minute. So when the machines work simultaneously, the rate at which the chemical is produced is the sum of these two rates, which is $\dfrac{k}{10} + \dfrac{k}{15} = k\left(\dfrac{1}{10} + \dfrac{1}{15}\right) = k\left(\dfrac{25}{150}\right) = \dfrac{k}{6}$ liters per minute. To compute the time required to produce k liters at this rate, divide the amount k by the rate $\dfrac{k}{6}$ to get $\dfrac{k}{\frac{k}{6}} = 6$.

Therefore, the correct answer is 6 minutes (or equivalent).

One way to check that the answer of 6 minutes is reasonable is to observe that if the slower rate of machine B were the same as machine A's faster rate of k liters in 10 minutes, then the two machines, working simultaneously, would take half the time, or 5 minutes, to produce the k liters. So the answer has to be <u>greater than 5 minutes</u>. Similarly, if the faster rate of machine A were the same as machine B's slower rate of k liters in 15 minutes, then the two machines would take half the time, or 7.5 minutes, to produce the k liters. So the answer has to be <u>less than 7.5 minutes</u>. Thus, the answer of 6 minutes is reasonable compared to the lower estimate of 5 minutes and the upper estimate of 7.5 minutes.

Strategy 2: Translate from Words to a Figure or Diagram

To solve a problem in which a figure is described but not shown, draw your own figure. Draw the figure as accurately as possible, labeling as many parts as possible, including any unknowns.

Drawing figures can help in geometry problems as well as in other types of problems. For example, in probability and counting problems, drawing a diagram can sometimes make it easier to analyze the relevant data and to notice relationships and dependencies.
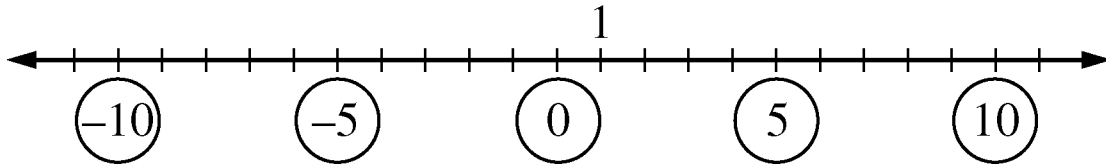
<u>Sample Question for Strategy 2: Multiple-Choice – Select One Answer Choice Question.</u>

Which of the following numbers is farthest from the number 1 on the number line?

Ⓐ   −10

Ⓑ   −5

Ⓒ   0

Ⓓ   5

Ⓔ   10

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Explanation

Circling each of the answer choices in a sketch of the following number line shows that of the given numbers, -10 is the greatest distance from 1.
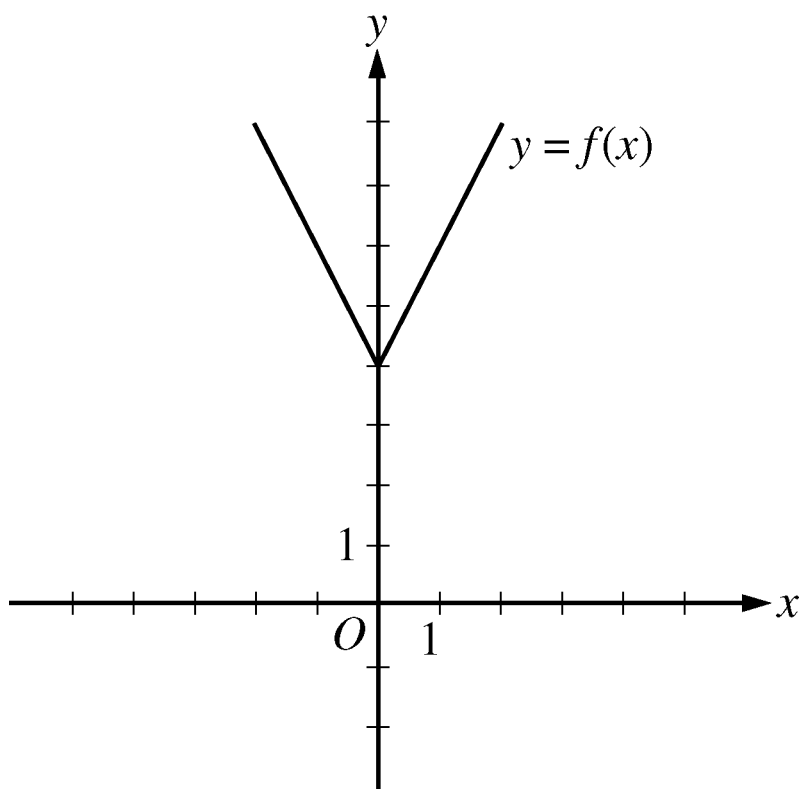

Another way to answer the question is to remember that the distance between two numbers on the number line is equal to the absolute value of the difference of the two numbers. For example, the distance between -10 and 1 is $|-10-1|=11$, and the distance between 10 and 1 is $|10-1|=|9|=9$. **The correct answer is Choice A, -10.**

Strategy 3: Translate from an Algebraic to a Graphical Representation

Many algebra problems can be represented graphically in a coordinate system, whether the system is a number line if the problem involves one variable, or a coordinate plane if the problem involves two variables. Such graphs can clarify relationships that may be less obvious in algebraic presentations.

<u>Sample Question for Strategy 3: Multiple-Choice – Select One Answer Choice Question.</u>

This question is based on the following figure.

$y = f(x)$

The figure shows the graph of the function f, defined by $f(x) = |2x| + 4$ for all numbers x. For which of the following functions g, defined for all numbers x, does the graph of g intersect the graph of f ?

(A) $g(x) = x - 2$
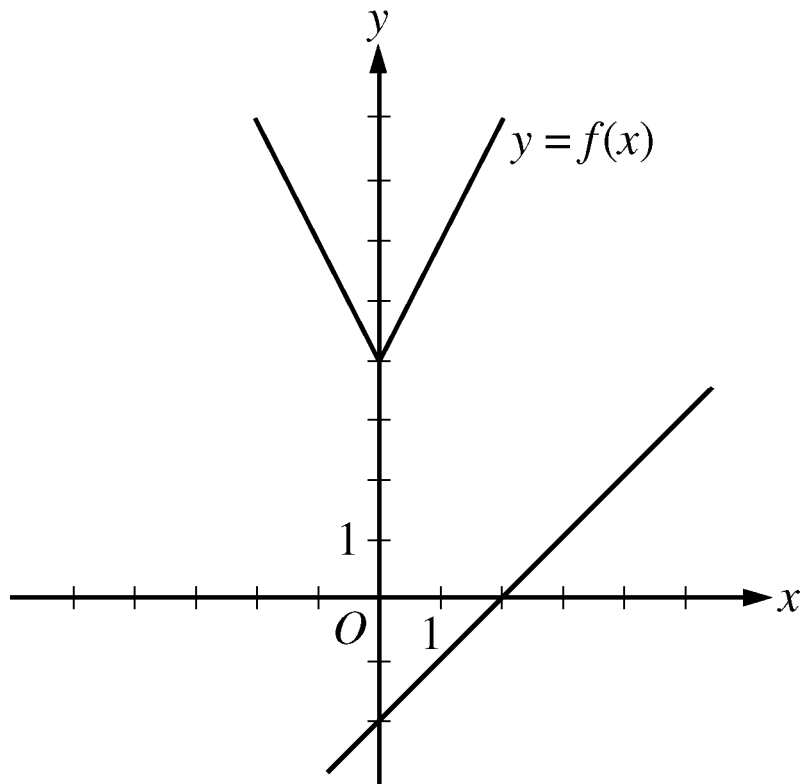
(B) $g(x) = x + 3$

(C) $g(x) = 2x - 2$

(D) $g(x) = 2x + 3$

(E) $g(x) = 3x - 2$

Explanation

You can see that all five choices are linear functions whose graphs are lines with various slopes and y-intercepts. The graph of Choice A is a line with slope 1 and y-intercept -2 shown in the following figure.

It is clear that this line will not intersect the graph of f to the left of the y-axis. To the right of the y-axis, the graph of f is a line with slope 2, which is greater than slope 1. Consequently, as the value of x increases, the value of y increases faster for f than for g, and therefore the graphs do not intersect to the right of the y-axis. Choice B is similarly ruled out. Note that if the y-intercept of either of the lines in choices A and B were greater than or equal to 4 instead of less than 4, they would intersect the graph of f.

Choices C and D are lines with slope 2 and *y*-intercepts less than 4. Hence, they are parallel to the graph of *f* (to the right of the *y*-axis) and therefore will not intersect it. Any line with a slope greater than 2 and a *y*-intercept less than 4, like the line in Choice E, will intersect the graph of *f* (to the right of the *y*-axis). **The correct answer is Choice E**, $g(x) = 3x - 2$.
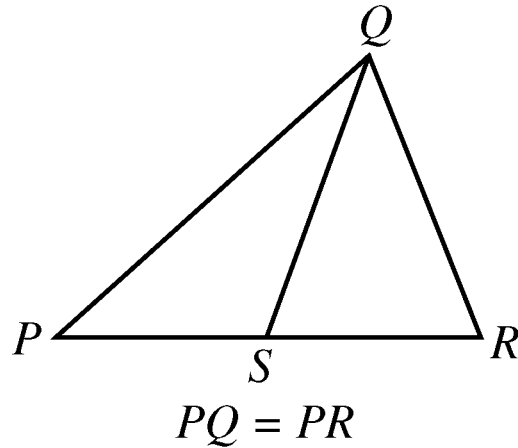
Note: This sample question also appears as a sample question for Strategy 6.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

When a figure is given in a problem, it may be effective to express relationships among the various parts of the figure using arithmetic or algebra.

Sample Question 1 for Strategy 4: Quantitative Comparison Question.

This question is based on the following figure.

$$PQ = PR$$

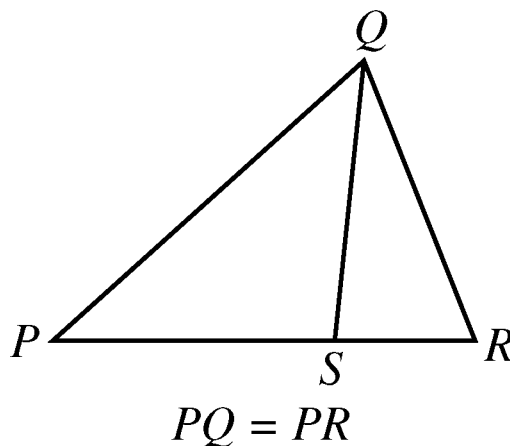| Quantity A	Quantity B
PS	SR

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

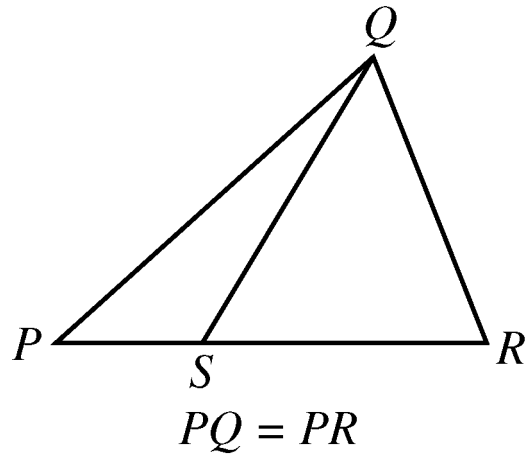(D) The relationship cannot be determined from the information given.

Explanation

From the figure accompanying the question, you know that *PQR* is a triangle and that point *S* is between points *P* and *R*, so $PS < PR$ and $SR < PR$. You are also given that $PQ = PR$. However, this information is not sufficient to compare *PS* and *SR*. Furthermore, because the figure is not necessarily drawn to scale, you cannot determine the relative sizes of *PS* and *SR* visually from the figure, though they may appear to be equal. The position of *S* can vary along side *PR* anywhere between *P* and *R*. Below are two possible variations of the figure accompanying the question, each of which is drawn to be consistent with the information $PQ = PR$.

Variation 1

$$PQ = PR$$

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Variation 2

$PQ = PR$

Note that Quantity A is greater in Variation 1 and Quantity B is greater in Variation 2. **Thus, the correct answer is Choice D, the relationship cannot be determined from the information given**.

Sample Question 2 for Strategy 4: Numeric Entry Question.

This question is based on the following table.

Results of a Used-Car Auction

	Small Cars	Large Cars
Number of cars offered	32	23
Number of cars sold	16	20
Projected sales total for cars offered (in thousands)	$70	$150
Actual sales total (in thousands)	$41	$120

For the large cars sold at an auction that is summarized in the table, what was the average sale price per car?

$								
	−	.	.	.	.	.	.	.
		0	0	0	0	0	0	0
		1	1	1	1	1	1	1
		2	2	2	2	2	2	2
		3	3	3	3	3	3	3
		4	4	4	4	4	4	4
		5	5	5	5	5	5	5
		6	6	6	6	6	6	6
		7	7	7	7	7	7	7
		8	8	8	8	8	8	8
		9	9	9	9	9	9	9

Explanation

From the table accompanying the question, you see that the number of large cars sold was 20 and the sales total for large cars was $120,000 (not $120). Thus the average sale price per car was $\dfrac{\$120,000}{20} = \$6,000$. **The correct answer is $6,000 (or equivalent)**.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

Arithmetic and algebraic representations include both expressions and equations. Your facility in simplifying a representation can often lead to a quick solution. Examples include converting from a percent to a decimal, converting from one measurement unit to another, combining like terms in an algebraic expression, and simplifying an equation until its solutions are evident.

Sample Question 1 for Strategy 5: Quantitative Comparison Question.

$$y > 4$$

Quantity A	Quantity B
$\dfrac{3y+2}{5}$	y

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Explanation

Set up the initial comparison:

$$\frac{3y+2}{5} \boxed{?} \ y$$

Then simplify:

Step 1: Multiply both sides by 5 to get $3y+2 \boxed{?} \ 5y$

Step 2: Subtract $3y$ from both sides to get $2 \boxed{?} \ 2y$

Step 3: Divide both sides by 2 to get $1 \boxed{?} \ y$

The comparison is now simplified as much as possible. In order to compare 1 and y, note that you are given the information $y > 4$ (above Quantities A and B). It follows from $y > 4$ that $y > 1$, or $1 < y$, so that in the comparison $1 \boxed{?} \ y$, the placeholder $\boxed{?}$ represents <u>less than</u> $(<)$: $1 < y$.

However, the problem asks for a comparison between Quantity A and Quantity B, not a comparison between 1 and y. To go from the comparison between 1 and y to a comparison between Quantities A and B, start with the last comparison, $1 < y$, and carefully consider each simplification step in reverse order to determine what each comparison implies about the preceding comparison, all the way back to the comparison between Quantities A and B if possible.

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Since step 3 was "<u>divide</u> both sides by 2," <u>multiplying</u> both sides of the comparison $1 < y$ by 2 implies the preceding comparison $2 < 2y$, thus reversing step 3. Each simplification step can be reversed as follows:

- Reverse step 3: <u>multiply</u> both sides by 2.
- Reverse step 2: <u>add</u> $3y$ to both sides.
- Reverse step 1: <u>divide</u> both sides by 5.

When each step is reversed, the relationship remains <u>less than</u> $(<)$, so Quantity A is less than Quantity B. **Thus, the correct answer is Choice B**, **Quantity B is greater**.

Sample Question 2 for Strategy 5: Numeric Entry Question.

A merchant made a profit of $5 on the sale of a sweater that cost the merchant $15. What is the profit expressed as a percent of the merchant's cost?

Give your answer to the nearest whole percent.

								%
−	.	.	.	.	.	.	.	
	0	0	0	0	0	0	0	
	1	1	1	1	1	1	1	
	2	2	2	2	2	2	2	
	3	3	3	3	3	3	3	
	4	4	4	4	4	4	4	
	5	5	5	5	5	5	5	
	6	6	6	6	6	6	6	
	7	7	7	7	7	7	7	
	8	8	8	8	8	8	8	
	9	9	9	9	9	9	9	

Explanation

The percent profit is $\left(\dfrac{5}{15}\right)(100) = 33.333... = 33.\overline{3}$ percent, which

is 33%, to the nearest whole percent. **Thus, the correct answer is 33% (or equivalent)**.

If you are taking the standard computer-based version of the test, and you use the calculator and the Transfer Display button, the number that will be transferred to the answer box is 33.333333, which is incorrect since it is not given to the nearest whole percent. You will need to adjust the number in the answer box by deleting all of the digits to the right of the decimal point (using the Backspace key).
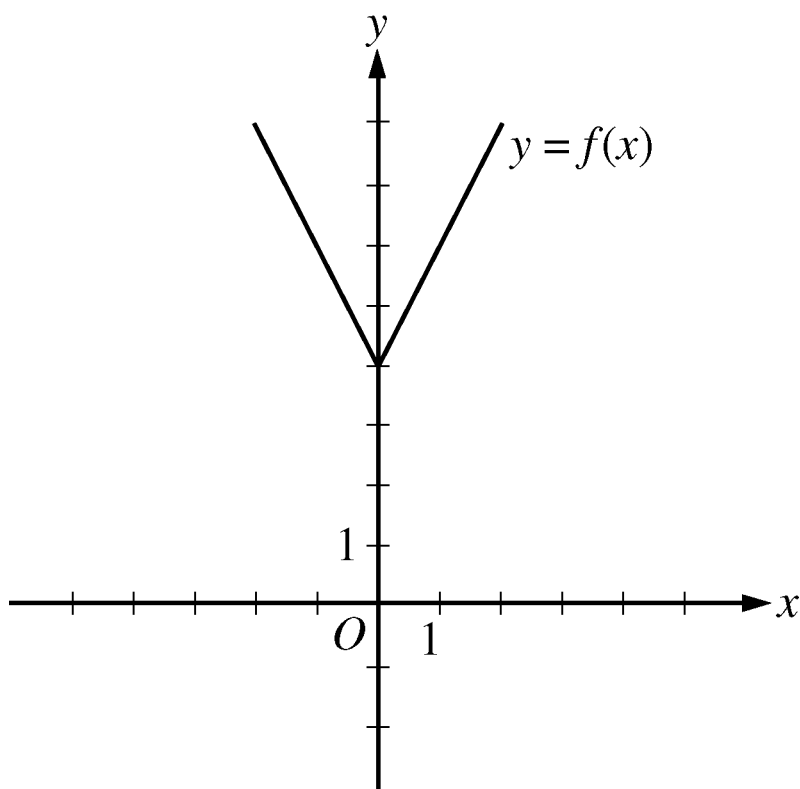
Also, since you are asked to give the answer as a percent, the decimal equivalent of 33 percent, which is 0.33, is incorrect. The percent symbol next to the answer box indicates that the form of the answer must be a percent. Entering 0.33 in the box would give the erroneous answer 0.33%.

Strategy 6: Add to a Geometric Figure

Sometimes you can add useful lines, points, or circles to a geometric figure to facilitate solving a problem. You can also add any given information—as well as any new information as you derive it—to the figure to help you see relationships within the figure more easily, for example, the length of a line segment or the measure of an angle.

<u>Sample Question 1 for Strategy 6: Multiple-Choice – Select One Answer Choice Question.</u>

This question is based on the following figure.

The figure shows the graph of the function f, defined by $f(x)=|2x|+4$ for all numbers x. For which of the following functions g, defined for all numbers x, does the graph of g intersect the graph of f ?
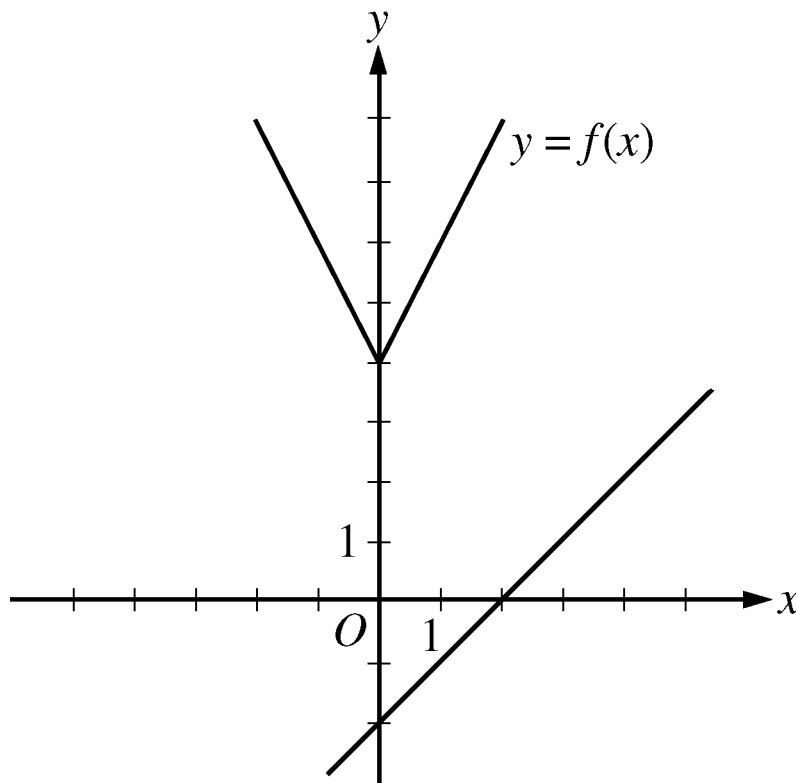
(A) $g(x)=x-2$

(B) $g(x)=x+3$

(C) $g(x)=2x-2$

(D) $g(x)=2x+3$

(E) $g(x)=3x-2$

Explanation

You can see that all five choices are linear functions whose graphs are lines with various slopes and y-intercepts. The graph of Choice A is a line with slope 1 and y-intercept -2 shown in the following figure.

It is clear that this line will not intersect the graph of f to the left of the y-axis. To the right of the y-axis, the graph of f is a line with slope 2, which is greater than slope 1. Consequently, as the value of x increases, the value of y increases faster for f than for g, and therefore the graphs do not intersect to the right of the y-axis. Choice B is similarly ruled out. Note that if the y-intercept of either of the lines in choices A and B were greater than or equal to 4 instead of less than 4, they would intersect the graph of f.

Choices C and D are lines with slope 2 and y-intercepts less than 4. Hence, they are parallel to the graph of f (to the right of the y-axis) and therefore will not intersect it. Any line with a slope greater than 2 and a y-intercept less than 4, like the line in Choice E, will intersect the graph of f (to the right of the y-axis). **The correct answer is Choice E**, $g(x) = 3x - 2$.

Note: This sample question also appears as a sample question for Strategy 3.

Strategy 7: Find a Pattern

Patterns are found throughout mathematics. Identifying a pattern is often the first step in understanding a complex mathematical situation. Pattern recognition yields insight that may point in the direction of a complete solution to the problem or simply help you generate a hypothesis, which requires further exploration using another strategy. In a problem where you suspect there is a pattern but don't recognize it yet, working with particular instances can help you identify the pattern. Once a pattern is identified, it can be used to answer questions.

Sample Question for Strategy 7: Multiple-Choice – Select One or More Answer Choices Question.

Which of the following could be the units digit of 57^n where n is a positive integer?

Indicate <u>all</u> such digits.

- A   0
- B   1
- C   2
- D   3
- E   4
- F   5
- G   6
- H   7
- I   8
- J   9

Explanation

The units digit of 57^n is the same as the units digit of 7^n for all positive integers n. To see why this is true for $n = 2$, compute 57^2 by hand and observe how its units digit results from the units digit of 7^2. Because this is true for every positive integer n, you need to consider only powers of 7. Beginning with $n = 1$ and proceeding consecutively, the units digits of 7, 7^2, 7^3, 7^4, and 7^5 are 7, 9, 3, 1, and 7, respectively. In this sequence, the first digit, 7, appears again, and the pattern of four digits, 7, 9, 3, 1, repeats without end. Hence, these four digits are the only possible units digits of 7^n and therefore of 57^n. **The correct answer consists of Choices B (1), D (3), H (7), and J (9)**.

Note: This sample question also appears as a sample question for Strategy 12.

Strategy 8: Search for a Mathematical Relationship

More general than patterns, mathematical relationships exist throughout mathematics. Problems may involve quantities that are related algebraically, sets that are related logically, or figures that are related geometrically. Also, there may be relationships between information given textually, algebraically, graphically, etc. To express relationships between quantities, it is often helpful to introduce one or more variables to represent the quantities. Once a relationship is understood and expressed, it is often the key to solving a problem.

Sample Question 1 for Strategy 8: Quantitative Comparison Question.

Quantity A	Quantity B
$x^2 + 1$	$2x - 1$

Ⓐ Quantity A is greater.

Ⓑ Quantity B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the information given.

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Explanation

Set up the initial comparison:

$$x^2 + 1 \;\boxed{?}\; 2x - 1$$

Then simplify by noting that the quadratic polynomial $x^2 - 2x + 1$ can be factored:

Step 1: Subtract $2x$ from both sides to get $x^2 - 2x + 1 \;\boxed{?}\; -1$

Step 2: Factor the left-hand side to get $(x - 1)^2 \;\boxed{?}\; -1$

The left-hand side of the comparison is the square of a number. Since the square of a number is always greater than or equal to 0, and 0 is greater than -1, the simplified comparison is the inequality $(x - 1)^2 > -1$ and the resulting relationship is <u>greater than</u> $(>)$. In reverse order, each simplification step implies the inequality <u>greater than</u> $(>)$ in the preceding comparison. Therefore, Quantity A is greater than Quantity B. **The correct answer is Choice A, Quantity A is greater**.

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Sample Question 2 for Strategy 8: Multiple-Choice – Select One or More Answer Choices Question.

Each employee of a certain company is in either Department X or Department Y, and there are more than twice as many employees in Department X as in Department Y. The average (arithmetic mean) salary is $25,000 for the employees in Department X and $35,000 for the employees in Department Y. Which of the following amounts could be the average salary for all of the employees of the company?

Indicate <u>all</u> such amounts.

A $26,000

B $28,000

C $29,000

D $30,000

E $31,000

F $32,000

G $34,000

Explanation

One strategy for answering this kind of question is to find the least and/or greatest possible value. Clearly the average salary is between $25,000 and $35,000, and all of the answer choices are in this interval. Since you are told that there are more employees with the lower average salary, the average salary of all employees must be less than the average of $25,000 and $35,000, which is $30,000. If there were exactly twice as many employees in Department X as in Department Y, then the average salary for all employees would be, to the nearest dollar, the following weighted mean,

$$\frac{(2)(25,000)+(1)(35,000)}{2+1} \approx 28,333 \text{ dollars}$$

where the weight for $25,000 is 2 and the weight for $35,000 is 1. Since there are <u>more</u> than twice as many employees in Department X as in Department Y, the actual average salary must be even closer to $25,000 because the weight for $25,000 is greater than 2. This means that $28,333 is the greatest possible average. Among the choices given, the possible values of the average are therefore $26,000 and $28,000. **Thus, the correct answer consists of Choices A ($26,000) and B ($28,000)**.

Intuitively, you might expect that any amount between $25,000 and $28,333 is a possible value of the average salary. To see that $26,000 is possible, in the weighted mean above, use the respective weights 9 and 1 instead of 2 and 1. To see that $28,000 is possible, use the respective weights 7 and 3.

Note: This sample question also appear as a sample question for Strategy 12.

Strategy 9: Estimate

Sometimes it is not necessary to perform extensive calculations to solve a problem—it is sufficient to estimate the answer. The degree of accuracy needed depends on the particular question being asked. Care should be taken to determine how far off your estimate could possibly be from the actual answer to the question. Estimation can also be used to check whether the answer to a question is reasonable.

<u>Sample Question 1 for Strategy 9: Quantitative Comparison Question.</u>

Quantity A	Quantity B
54% of 360	150

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Explanation

Without doing the exact computation, you can see that 54 percent of 360 is greater than $\frac{1}{2}$ of 360, which is 180, and 180 is greater than Quantity B, 150. **Thus, the correct answer is Choice A, Quantity A is greater**.

Sample Question 2 for Strategy 9: Multiple-Choice – Select One Answer Choice Question.

A car got 33 miles per gallon using gasoline that cost $2.95 per gallon. Approximately what was the cost, in dollars, of the gasoline used in driving the car 350 miles?

(A) $10

(B) $20

(C) $30

(D) $40

(E) $50

Explanation

Scanning the answer choices indicates that you can do at least some estimation and still answer confidently. The car used $\frac{350}{33}$ gallons of gasoline, so the cost was $\left(\frac{350}{33}\right)(2.95)$ dollars. You can estimate the product $\left(\frac{350}{33}\right)(2.95)$ by estimating $\frac{350}{33}$ a little low, 10, and estimating 2.95 a little high, 3, to get approximately $(10)(3) = 30$ dollars. You can also use the calculator to compute a more exact answer and then round the answer to the nearest 10 dollars, as suggested by the answer choices. The calculator yields the decimal 31.287…, which rounds to 30 dollars. **Thus, the correct answer is Choice C, $30**.

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Strategy 10: Trial and Error

Version 1: Make a Reasonable Guess and then Refine It

For some problems, the fastest way to a solution is to make a reasonable guess at the answer, check it, and then improve on your guess. This is especially useful if the number of possible answers is limited. In other problems, this approach may help you at least to understand better what is going on in the problem.

Sample Question for Strategy 10, Version 1: Multiple-Choice – Select One Or More Answer Choice Question.

Which two of the following numbers have a product that is between −1 and 0 ?

Indicate <u>both</u> of the numbers.

A −20

B −10

C 2^{-4}

D 3^{-2}

Explanation

For this question, you must select a pair of answer choices. The product of the pair must be negative, so the possible products are $(-20)\left(2^{-4}\right)$, $(-20)\left(3^{-2}\right)$, $(-10)\left(2^{-4}\right)$, and $(-10)\left(3^{-2}\right)$. The product must also be greater than -1. The first product is $\dfrac{-20}{2^4} = -\dfrac{20}{16} < -1$, the second product is $\dfrac{-20}{3^2} = -\dfrac{20}{9} < -1$, and the third product is, $\dfrac{-10}{2^4} = -\dfrac{10}{16} > -1$, so you can stop there. **The correct answer consists of Choices B (-10) and C $\left(2^{-4}\right)$.**

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Version 2: Try More Than One Value of a Variable

To explore problems containing variables, it is useful to substitute values for the variables. It often helps to substitute more than one value for each variable. How many values to choose and what values are good choices depends on the problem. Also dependent on the problem is whether this approach, by itself, will yield a solution or whether the approach will simply help you generate a hypothesis that requires further exploration using another strategy.

Sample Question 1 for Strategy 10, Version 2: Quantitative Comparison Question.

Lionel is younger than Maria.

Quantity A	Quantity B
Twice Lionel's age	Maria's age

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

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Explanation

If Lionel's age is 6 years and Maria's age is 10 years, then Quantity A is greater, but if Lionel's age is 4 years and Maria's age is 10 years, then Quantity B is greater. Thus, the relationship cannot be determined. **The correct answer is Choice D, the relationship cannot be determined from the information given**.

Sample Question 2 for Strategy 10, Version 2: Quantitative Comparison Question.

$$y = 2x^2 + 7x - 3$$

Quantity A	Quantity B
x	y

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Explanation

If $x = 0$ then $y = 2(0^2) + 7(0) - 3 = -3$ so in this case, $x > y$; but

if $x = 1$, then $y = 2(1^2) + 7(1) - 3 = 6$, so in that case, $y > x$. **Thus, the correct answer is Choice D, the relationship cannot be determined from the information given.**

Note that plugging numbers into expressions <u>may not</u> be conclusive. However, it <u>is</u> conclusive if you get different results after plugging in different numbers: the conclusion is that the relationship cannot be determined from the information given. It is also conclusive if there are only a small number of possible numbers to plug in and <u>all</u> of them yield the same result, say, that Quantity B is greater.

Now suppose there are an infinite number of possible numbers to plug in. If you plug many of them in and each time the result is, for example, that Quantity A is greater, you still cannot conclude that Quantity A is greater for every possible number that could be plugged in. Further analysis would be necessary and should focus on whether Quantity A is greater for all possible numbers or whether there are numbers for which Quantity A is not greater.

Strategy 11: Divide into Cases

Some problems are quite complex. To solve such problems you may need to divide them into smaller, less complex problems, which are restricted cases of the original problem. When you divide a problem into cases, you should consider whether or not to include all possibilities. For example, if you want to prove that a certain statement is true for all integers, it may be best to show that it is true for all positive integers, then show it is true for all negative integers, and then show it is true for zero. In doing that, you will have shown that the statement is true for all integers, because each integer is either positive, negative, or zero.

Sample Question 1 for Strategy 11: Quantitative Comparison Question.

Quantity A	Quantity B
The least prime number greater than 24	The greatest prime number less than 28

Ⓐ Quantity A is greater.

Ⓑ Quantity B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the information given.

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Explanation

For the integers greater than 24, note that 25, 26, 27, and 28 are not prime numbers, but 29 is a prime number, as are 31 and many other greater integers. Thus, 29 is the least prime number greater than 24, and Quantity A is 29. For the integers less than 28, note that 27, 26, 25, and 24 are not prime numbers, but 23 is a prime number, as are 19 and several other lesser integers. Thus, 23 is the greatest prime number less than 28, and Quantity B is 23. **Thus, the correct answer is Choice A, Quantity A is greater.**

Sample Question 2 for Strategy 11: Multiple-Choice – Select One or More Answer Choices Question.

Which of the following integers are multiples of both 2 and 3?

Indicate <u>all</u> such integers.

A 8

B 9

C 12

D 18

E 21

F 36

Explanation

You can first identify the multiples of 2, which are 8, 12, 18 and 36, and then among the multiples of 2 identify the multiples of 3, which are 12, 18 and 36. Alternatively, if you realize that every number that is a multiple of 2 and 3 is also a multiple of 6, you can identify the choices that are multiples of 6. **The correct answer consists of Choices C (12), D (18), and F (36)**.

Strategy 12: Adapt Solutions to Related Problems

When solving a new problem that seems similar to a problem that you know how to solve, you can try to solve the new problem by adapting the solution—both the strategies and the results—of the problem you know how to solve.

If the differences between the new problem and the problem you know how to solve are only surface features—for example, different numbers, different labels, or different categories—that is, features that are not fundamental to the structure of the problem, then solve the new problem using the same strategy as you used before.

If the differences between the new problem and the problem you know how to solve are more than just surface features, try to modify the solution to the problem you know how to solve to fit the conditions given in the new problem.

Sample Question 1 for Strategy 12: Multiple-Choice – Select One or More Answer Choices Question.

Each employee of a certain company is in either Department X or Department Y, and there are more than twice as many employees in Department X as in Department Y. The average (arithmetic mean) salary is $25,000 for the employees in Department X and $35,000 for the employees in Department Y. Which of the following amounts could be the average salary for all of the employees of the company?

Indicate <u>all</u> such amounts.

A $26,000

B $28,000

C $29,000

D $30,000

E $31,000

F $32,000

G $34,000

Explanation

One strategy for answering this kind of question is to find the least and/or greatest possible value. Clearly the average salary is between $25,000 and $35,000, and all of the answer choices are in this interval. Since you are told that there are more employees with the lower average salary, the average salary of all employees must be less than the average of $25,000 and $35,000, which is $30,000. If there were exactly twice as many employees in Department X as in Department Y, then the average salary for all employees would be, to the nearest dollar, the following weighted mean,

$$\frac{(2)(25,000)+(1)(35,000)}{2+1} \approx 28,333 \text{ dollars}$$

where the weight for $25,000 is 2 and the weight for $35,000 is 1. Since there are <u>more</u> than twice as many employees in Department X as in Department Y, the actual average salary must be even closer to $25,000 because the weight for $25,000 is greater than 2. This means that $28,333 is the greatest possible average. Among the choices given, the possible values of the average are therefore $26,000 and $28,000. **Thus, the correct answer consists of Choices A ($26,000) and B ($28,000)**.

Intuitively, you might expect that any amount between $25,000 and $28,333 is a possible value of the average salary. To see that $26,000 is possible, in the weighted mean above, use the respective weights 9 and 1 instead of 2 and 1. To see that $28,000 is possible, use the respective weights 7 and 3.

Note: This sample question also appears as a sample question for Strategy 8.

Sample Question 2 for Strategy 12: Multiple-Choice – Select One or More Answer Choices Question.

Which of the following could be the units digit of 57^n where n is a positive integer?

Indicate all such digits.

A 0

B 1

C 2

D 3

E 4

F 5

G 6

H 7

I 8

J 9

Explanation

The units digit of 57^n is the same as the units digit of 7^n for all positive integers n. To see why this is true for $n = 2$, compute 57^2 by hand and observe how its units digit results from the units digit of 7^2. Because this is true for every positive integer n, you need to consider only powers of 7. Beginning with $n = 1$ and proceeding consecutively, the units digits of 7, 7^2, 7^3, 7^4, and 7^5 are 7, 9, 3, 1, and 7, respectively. In this sequence, the first digit, 7, appears again, and the pattern of four digits, 7, 9, 3, 1, repeats without end. Hence, these four digits are the only possible units digits of 7^n and therefore of 57^n. **The correct answer consists of Choices B (1), D (3), H (7), and J (9)**.

Note: This sample question also appears as a sample question for Strategy 7.

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Strategy 13: Determine Whether a Conclusion Follows from the Information Given

In some problems, you are given information and a statement describing a possible conclusion, which may or may not follow from the information. You need to determine whether or not the conclusion is a logical consequence of the information given.

If you think that the conclusion follows from the information, try to show it. Using the information and any relevant mathematical relationships, try to reason step-by-step from the information to the conclusion. Another way to show that the conclusion follows from the information, is to show that in <u>all</u> cases in which the information is true, the conclusion is also true.

If you think that the conclusion does <u>not</u> follow from the information, try to show that instead. One way to show that a conclusion does not follow from the information is to produce a counterexample. A counterexample is a case where the given information is true but the conclusion is false. If you are unsuccessful in producing a counterexample, it does not necessarily mean that the conclusion does not follow from the information—it may mean that although a counterexample exists, you were not successful in finding it.

Sample Question 1 for Strategy 13: Quantitative Comparison Question.

$$w > 1$$

Quantity A	Quantity B
$7w - 4$	$2w + 5$

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Explanation

Set up the initial comparison:

$$7w - 4 \;\boxed{?}\; 2w + 5$$

Then simplify:

Step 1: Subtract $2w$ from both sides and add 4 to both sides to get

$$5w \;\boxed{?}\; 9$$

Step 2: Divide both sides by 5 to get

$$w \;\boxed{?}\; \frac{9}{5}$$

The comparison cannot be simplified any further. Although you are given $w > 1$, that you still don't know how w compares to $\frac{9}{5}$, or 1.8. For example, if $w = 1.5$, then $w < 1.8$, but if $w = 2$, then $w > 1.8$. In other words, the relationship between w and $\frac{9}{5}$ cannot be determined.

Note that each of these simplification steps is reversible, so in reverse order, each simplification step implies that the <u>relationship cannot be determined</u> in the preceding comparison. Thus, the relationship between Quantities A and B cannot be determined. **The correct answer is Choice D**, **the relationship cannot be determined from the information given**.

Sample Question 2 for Strategy 13: Multiple-Choice – Select One or More Answer Choices Question.

This question is based on the following table.

Annual Percent Change in Dollar Amount of Sales at Five Retail Stores from 2006 to 2008

Store	Percent Change from 2006 to 2007	Percent Change from 2007 to 2008
P	10	−10
Q	−20	9
R	5	12
S	−7	−15
T	17	−8

Based on the information given, which of the following statements must be true?

Indicate <u>all</u> such statements.

A For 2008 the dollar amount of sales at Store R was greater than that at each of the other four stores.

B The dollar amount of sales at Store S for 2008 was 22 percent less than that for 2006.

C The dollar amount of sales at Store R for 2008 was more than 17 percent greater than that for 2006.

Explanation

For Choice A, since the only data given in the table accompanying the question are percent changes from year to year, there is no way to compare the actual dollar amount of sales at the stores for 2008 or for any other year. Even though Store R had the greatest percent increase from 2006 to 2008, its actual dollar amount of sales for 2008 may have been much smaller than that for any of the other four stores, and therefore Choice A is not necessarily true.

For Choice B, even though the sum of the two percent decreases would suggest a 22 percent decrease, the bases of the percents are different. If B is the dollar amount of sales at Store S for 2006, then the dollar amount for 2007 is 93 percent of B, or $0.93B$, and the dollar amount for 2008 is given by $(0.85)(0.93)B$, which is $0.7905B$. Note that this represents a percent decrease of $100 - 79.05 = 20.95$ percent, which is not equal to 22 percent, and so Choice B is not true.

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For Choice C, if C is the dollar amount of sales at Store R for 2006, then the dollar amount for 2007 is given by $1.05C$ and the dollar amount for 2008 is given by $(1.12)(1.05)C$, which is $1.176C$. Note that this represents a 17.6 percent increase, which is greater than 17 percent, so Choice C must be true.

Therefore, the correct answer consists of only Choice C (The dollar amount of sales at Store R for 2008 was more than 17 percent greater than that for 2006).

Note: This sample question also appears as a sample question for Strategy 14.

Strategy 14: Determine What Additional Information Is Sufficient to Solve a Problem

Some problems cannot be solved directly from the information given, and you need to determine what other information will help you answer the question. In that case, it is useful to list all the information given in the problem, along with the information that would be contained in a complete solution, and then evaluate what is missing. Sometimes the missing information can be derived from the information given, and sometimes it cannot.

Sample Question for Strategy 14: Multiple-Choice – Select One or More Answer Choices Question.

This question is based on the following table.

Annual Percent Change in Dollar Amount of Sales at Five Retail Stores from 2006 to 2008

Store	Percent Change from 2006 to 2007	Percent Change from 2007 to 2008
P	10	–10
Q	–20	9
R	5	12
S	–7	–15
T	17	–8

Based on the information given, which of the following statements must be true?

Indicate all such statements.

[A] For 2008 the dollar amount of sales at Store *R* was greater than that at each of the other four stores.

[B] The dollar amount of sales at Store *S* for 2008 was 22 percent less than that for 2006.

[C] The dollar amount of sales at Store *R* for 2008 was more than 17 percent greater than that for 2006.

Explanation

For Choice A, since the only data given in the table accompanying the question are percent changes from year to year, there is no way to compare the actual dollar amount of sales at the stores for 2008 or for any other year. Even though Store R had the greatest percent increase from 2006 to 2008, its actual dollar amount of sales for 2008 may have been much smaller than that for any of the other four stores, and therefore Choice A is not necessarily true.

For Choice B, even though the sum of the two percent decreases would suggest a 22 percent decrease, the bases of the percents are different. If B is the dollar amount of sales at Store S for 2006, then the dollar amount for 2007 is 93 percent of B, or $0.93B$, and the dollar amount for 2008 is given by $(0.85)(0.93)B$, which is $0.7905B$. Note that this represents a percent decrease of $100 - 79.05 = 20.95$ percent, which is not equal to 22 percent, and so Choice B is not true.

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For Choice C, if C is the dollar amount of sales at Store R for 2006, then the dollar amount for 2007 is given by $1.05C$ and the dollar amount for 2008 is given by $(1.12)(1.05)C$, which is $1.176C$. Note that this represents a 17.6 percent increase, which is greater than 17 percent, so Choice C must be true.

Therefore, the correct answer consists of only Choice C (The dollar amount of sales at Store R for 2008 was more than 17 percent greater than that for 2006).

<u>Note:</u> This sample question also appears as a sample question for Strategy 13.